The θ projection method and small creep strain interpolations in a commercial Titanium alloy

M. EVANS
Department of Materials Engineering, University of Wales Swansea, Singleton Park, Swansea, UK, SA2 8PP
E-mail: m.evans@swansea.ac.uk

The paper describes the early and final uniaxial creep behaviour of a Titanium alloy used for manufacturing intermediate power compressor disks and blades. Tests were conducted at the operating temperature (773 K) for such components and for rupture lives up to 3600 hours. Creep curves were fitted using either the conventional 4Θ model or the recently developed 6θ equation. Parameters allowing the interpolation of times to small strains were produced and their accuracy checked against experimental values using distributions found to be most supported by the data. At strains above 0.75% both methods yielded zero mean interpolation errors. At strains above 0.27% and below 0.75% the 4Θ equation produced systematic errors in interpolation but the 6θ function gave errors which were not statistically different from zero. For strains below 0.27% both techniques produced systematic interpolation errors but the 6θ interpolations were always significantly better than their 4Θ counterparts. Both the 6θ and 4Θ techniques produced systematic errors when predicting the failure time using interpolated rupture strains. Unlike the 4Θ function, the 6θ equation produced unbiased predictions of the minimum creep rate and so produced failure time interpolations from the Monkman–Grant relation that were indistinguishable from zero. © *2001 Kluwer Academic Publishers*

1. Introduction

The constant drive to produce lighter aero engines lead, in the late 1980's and early 1990's, to the development of some new Titanium alloys, such as IMI 834 and Ti 6.2.4.6. Unlike Ti-4Sn-4Al-4Mo-0.5Si (IMI 551) [1] which must operate at temperatures below 723 K, both of these new alloys are capable of operating in the 773 K to 900 K temperature range [2]. As a result these alloys are now being used in the manufacture of intermediate to high power (as opposed to the traditional low power role of Titanium) compressor disks and blades for modern aero engines.

However, at these higher operating temperatures conditions are ideal for creep at significant rates and so it is very important to have a detailed understanding of the creep behaviour of the Ti 6.2.4.6 alloy. In particular, compressor disks and blades are made to close dimensional tolerances and so must operate without to much distortion. Strains in excess of 0.5% over the creeping range of an aero engine blade can cause the surrounding casing to be spoiled. Designers therefore need to know the creep properties of the Ti 6.2.4.6 alloy at small creep strains but these are difficult to measure by conventional creep curves since it is as this time that creep machines are at their most unstable.

One measure of early creep properties is the time to a given small strain, such at 0.1%. It will normally be required to know such a time at the operating conditions (stress and temperature) experienced by the high power Titanium disks and blades so that interpolation (and possibly extrapolation) techniques will be required. The θ projection method, in its more usual form [3] has proved to be very useful for the interpolation of properties at large creep strains but unfortunately is much less reliable at small strains [4, 5]. This paper investigates the effectiveness of using a recent modification of the θ projection method [6] to obtain interpolative predictions of times to various low strains in a Ti 6.2.4.6 alloy.

To achieve such an aim this paper will first describe the test material and the test matrix used which tally's with the operating conditions experienced by a Ti 6.2.4.6 alloy aero engine disk/blade. The accuracy of the traditional and modified theta projection techniques (to be called 4Θ and 6θ respectively from now on) will then be analysed through the application of some statistical techniques to the observed errors in interpolation. General practical implications and conclusions will then be discussed.

2. Experimental procedures

The material used in this investigation is Titanium alloy Ti-6.2.4.6 prepared as an ingot forged in the β-phase at approximately 1238 K (the β transus is approximately 1223 K). The chemical composition of this material (in wt %) was determined as 5.79 Al, 3.94 Zr, 1.99 Sn, 6.03 Mo, 0.06 Fe, 0.02 C, 0.1 O,

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TABLE I Creep test matrix for the Titanium alloy Ti-6.2.4.6

τ	t_F(ks)	$\dot{\varepsilon}_M$(s^{-1})	ε_F (%)	$t_{0.05\%}$(ks)	$t_{0.1\%}$(ks)	$t_{0.2\%}$(ks)	$t_{0.5\%}$(ks)	$T_{1\%}$(ks)
480	12753	4.31E-09	7.4	1.02	4.27	20.2	175.2	925.8
535	6310	1.41E-08	17.95	0.97	2.41	7.42	50.53	251.46
560	3024	2.94E-08	14.75	0.25	1.08	4.34	29.6	122.51
580*	4277	2.09E-08	17.75	0.6	1.72	5.76	38.79	177.89
600	3643	2.81E-08	27.76	0.22	0.79	3.21	21.6	98.75
620	1725	5.61E-08	17.79	0.35	0.76	2.92	14.69	62.37
700	855	1.13E-07	17.24	0.35	0.66	1.3	5.25	20.74
800	255	5.30E-07	20.17	0.1	0.21	0.49	1.69	4.35
900	40	2.20E-06	12.98	0.01	0.04	0.11	0.42	1.22

All the above results were obtained at a temperature of 773 K. τ is the stress level, t_F the time to failure in thousands of seconds, $\dot{\varepsilon}_M$ the minimum creep rate in strain per second, ε_F the rupture strain in percent and $t_{x\%}$ time to $x\%$ strain in thousands of seconds.
*Properties shown are an arithmetic average of the results obtained from the fifteen specimens placed on test at this condition.

0.0035 Si and Balance ~82 Ti. The initial heat treatment schedule used was at 1173 K for two hours, following which the material was air quenched. It was then reheated to 868 K, held for eight hours, and finally air quenched. A second re-age for two hours at 913 K, in air, was employed, which is a simulated Post Weld Heat Treatment.

Twenty-three conventional creep specimens, from material supplied by TIMET U.K. Ltd., of 3.8 mm diameter, 25.4 mm gauge length and 3/8 inch BSF thread were machined from the heat treated material and tested in tension over a range of stresses at 773 K using seven high precision uniaxial constant-stress machines fitted with three zone furnaces. All machines had been calibrated to British Standards BSEN10002 (parts 1–5). Details of such testing machines can be found in most texts on creep [7]. Temperature was maintained along the gauge length and with respect to time to better than ±0.5 K and the extensometers were capable of establishing creep strains to better than 10^{-5} and the creep strain-time curves contained approximately 400 points.

The test matrix used is summarised in Table I. Fifteen of the twenty three specimens were placed on test at 773 K and a stress of 580 MPa, with the remaining eight specimens being tested at 773 K and over the stress range 900 MPa to 480 MPa—excluding 580 MPa. The creep properties contained in this Table have been reported (in part) elsewhere [8]. This test matrix was designed so that the creep property predictions made at these test conditions correspond to interpolations at the operating conditions experienced by an Ti 6.2.4.6 alloy aero engine disk or blade.

3. The θ projection concept: Old and new

3.1. 4Θ and 6θ models

Any θ projection technique has three basic steps. First, there is the experimental stage where uniaxial constant stress creep curves are measured over a range of stresses and temperatures. Second, the form of these creep curves are modelled in such a way that the form can be projected to other stresses and temperatures—either within (i.e. interpolation) or outside (i.e. extrapolation) the original range of test conditions. Finally, the required creep properties (such as the minimum creep rate or time to $x\%$ strain) are 'read off' the projected creep curves. This 'reading off' often requires the application of a suitable numerical technique.

A single creep curve at steady uniaxial stress τ and absolute temperature T can be modelled using a general functional form

$$\varepsilon = \eta(t, \Theta_1, \Theta_2, \ldots, \Theta_j, \ldots, \Theta_m), \qquad (1a)$$

where η is some non-linear function, ε is the uniaxial creep strain at time t and Θ_j are numerical parameters that can be determined from the experimental creep curves using a suitable estimation technique. For a series of experimental creep curves obtained under different test conditions, the Θ_j are related to τ and T by interpolation functions of the form,

$$\Theta_j = g_j(\tau, T, b_{j1}, b_{j2}, \ldots, b_{jk}, \ldots, b_{jp}), \qquad (1b)$$

where g_j are some linear or more likely non linear functions, j is a subscript identifying Θ in Equation 1a and b_{jk} are constants that need to be determined using a suitable estimation technique. Equation 1b permits the projection of Θ_j to new conditions of stress and temperature and hence the projection of the complete creep curve to those new conditions.

A variety of different equations have been used to describe the form of η in Equation 1a [9, 10]. The form that has been most used over more recent years and has been shown to give a good representation (at least for large strains) of an experimental creep curve [11] is

$$\varepsilon = \Theta_1(1 - e^{-\Theta_2 t}) + \Theta_3(e^{\Theta_4 t} - 1). \qquad (2a)$$

When taken together with the following representation for the function g_j in Equation 1b,

$$\ln(\Theta_j) = b_{j1} + b_{j2}\tau + b_{j3}T + b_{j4}\tau T, \qquad (2b)$$

the method has yielded excellent predictions of creep properties for moderate to large strains. Equations 2 corresponds to $m = p = 4$ and so projections obtained using Equations 2 will be termed 4Θ projections.

At small strains, the fit of Equation 2a to the experimental creep curve is very poor and so is misspecified. This misspecification has lead to considerable errors in the calculation of times to small strains and to initial and minimum creep rates in various steel and aluminium alloys [12]. A modification to the 4Θ model has been recently suggested [6] and applied [4, 5, 12]. It includes in combination with Equation 2b (with θ_j replacing Θ_j) an additional term in Equation 1a that is intended to deal specifically with early primary creep. This 6θ model (because it involves 6 theta terms) has the form

$$\varepsilon = \theta_1(1 - e^{-\theta_2 t}) + \theta_3(e^{\theta_4 t} - 1) + \theta_5(1 - e^{-\theta_6 t}). \quad (3)$$

Lower case letters are used merely to distinguish these theta parameters from those in Equation 2a. The first two terms on the right hand side have the same physical meaning as in Equation 2a; the first term describes normal primary creep and the second term normal tertiary creep. The third term is new and describes early primary behaviour. For some materials [5, 12] the fit to the experimental data is dramatically improved.

3.2. Estimation of and prediction from 4Θ and 6θ models

For both Equations 2a and 3, Θ_j and θ_j can be estimated from the experimental creep curves, obtained at the test conditions described in Section 2 above, by a non linear least squares procedure described elsewhere [11]. This procedure adjusts the estimates of Θ or θ, and more especially their standard errors, for 1st order autocorrelation in the scatter about the fitted curve. This procedure was implemented within a spreadsheet environment by making use of Excels Solver Function. This uses the Conjugate Gradient optimisation technique. Values of the constants in Equation 2b were determined from the estimated Θ_j or θ_j values using an unweighted linear least squares procedure. (This was again achieved in Excel's using its Linest Function).

The procedure used for estimating the time required for a particular strain is similar regardless of whether the 4Θ or 6θ model is used. At any arbitrary stress and temperature, Θ_j and/or θ_j are calculated from Equation 2b. The time, $t_{x\%}$, required to attain a strain of $\varepsilon_{x\%}$ is the numerical solution (using Newton – Ralphson methods) to the equation

$$0 = \Theta_1(1 - e^{-\Theta_2 t}) + \Theta_3(e^{\Theta_4 t} - 1) - \frac{\varepsilon_{x\%}}{100} \quad (4a)$$

for the 4Θ model or,

$$0 = \theta_1(1 - e^{-\theta_2 t}) + \theta_3(e^{-\theta_4 t} - 1) + \theta_5(1 - e^{-\theta_6 t}) - \frac{\varepsilon_{x\%}}{100} \quad (4b)$$

for the 6θ model. Replacing $\varepsilon_{x\%/100}$ with the rupture strain ε_F in Equations 4 and solving will give a prediction of the failure time, t_F. This approach to failure time prediction involves modelling the rupture strain using stress and temperature as explanatory variables through the following equation

$$\varepsilon_F = a_1 + a_2\tau + a_3 T + a_4 \tau T. \quad (4c)$$

Unfortunately, ε_F is not strongly dependant on stress or temperature. Instead it seems to vary randomly and substantially over various test conditions. Whilst this tends to lead to errors in failure time predictions it should be noted that this transmitted error is minimised by the fact that the creep curve is at its steepest around the time to failure.

An alternative approach to predicting the time to failure involves first predicting the minimum creep rate, $\dot{\varepsilon}_M$. A prediction of the minimum creep rate using the 4Θ model can be found from

$$t_M = \frac{1}{\Theta_2 + \Theta_4} \ln \frac{\Theta_1 \Theta_2^2}{\Theta_3 \Theta_4^2}, \quad (5a)$$

$$\dot{\varepsilon}_M = -\Theta_1 \Theta_2 e^{-\Theta_2 t_M} + \Theta_3 \Theta_4 e^{\Theta_4 t_M}. \quad (5b)$$

where t_M is time to minimum creep rate.

The minimum creep rate can also be predicted using the 6θ model by solving the following equation numerically

$$\frac{\theta_1 \theta_2^2}{\theta_3 \theta_4^2} e^{t[-\theta_2 - \theta_4]} + \frac{\theta_5 \theta_6^2}{\theta_3 \theta_4^2} e^{t[-\theta_6 - \theta_4]} - 1 = 0. \quad (5c)$$

Once a minimum creep rate has been predicted it can be used to predict the time to failure by making use of the Monkman–Grant [13] relation that stipulates that the time to failure is inversely proportional to the minimum creep rate,

$$t_F = \frac{\alpha}{\varepsilon_M^{\beta}}, \quad (6)$$

where α and β are constants that can be estimated from the test matrix data set using a least squares procedure.

3.3. Evaluating the accuracy of creep property predictions obtained from 4θ and 6θ models

It is sensible to analyse the difference between interpolated and experimental results in terms of the logarithms of the creep property being studied. If the experimental creep property (such as time to $x\%$ strain) is κ_e and the interpolated (i.e. predicted) property κ_p then

$$\ln(\kappa_e) = \ln(\kappa_p) + \upsilon, \quad (7a)$$

where υ was a random variable which can be estimated from the observed creep properties. The relevant prediction error is thus defined as

$$\upsilon = \ln(\kappa_e) - \ln(\kappa_p). \quad (7b)$$

When defined in this way, υ (for small υ) is equal to a proportionate prediction error, i.e. the error in predicting a creep property as a proportion of the actual creep property. As such it is reasonable to assume that υ will be independent of the numeric value for the creep property being predicted. υ will however contain several types of error. Some types will be due to statistical scatter in the creep measuring process and these errors are likely to have a population mean of zero. Others will be due to a possible mis specification of the creep curve model and or interpolation function and these are likely to have a population mean that is different from zero.

In a large sample, testing for such a mis specification (i.e. zero mean prediction errors) would be straightforward. The central limit theorem would ensure that a student t statistic test for a zero mean would be normally distributed, and so critical values could be obtained from a student t table. However, in this paper there are just twenty three prediction errors that can be calculated for a given creep property and so a different approach must be sought. In this paper a generalised three parameter log gamma distribution will be used to model the prediction errors, υ, as defined above.

The three parameter log gamma distribution takes the following form [14],

$$f(\upsilon_i) = \frac{|\lambda|}{\sigma\Gamma(\lambda^{-2})}(\lambda^{-2})^{\lambda-2}\exp\left[\lambda^{-2}(\lambda w_i - \exp(\lambda w_i)\right], \tag{8a}$$

where υ_i is the ith value for a creep property prediction error, $f(\upsilon_i)$ is the density function for υ, μ a location parameter, σ a scale parameter, λ a shape parameter and w_i the transformation $(\upsilon_i - \mu)/\sigma$.

When $\lambda = 0$, υ_i follows a normal distribution with a mean equal to μ and a variance equal to σ^2. ($\exp(\upsilon_i)$ is therefore log normally distributed). When $\lambda = 1$, υ_i follows an extreme value distribution with a mean equal to $\mu - 0.5772\sigma$ and a variance equal to $(\pi^2\sigma^2)/6$. ($\exp(\upsilon_i)$ is therefore Weibull distributed). The exponential and gamma distributions are also special cases of this generalised distribution. For any other value of λ, υ_i follows a three parameter gamma distribution with a mean equal to

$$\text{Mean}\,(\upsilon) = \mu + \sigma\lambda^{-1}[\psi(\lambda^{-2}) - \ln(\lambda^{-2})], \tag{8b}$$

and a variance equal to

$$\text{Variance}\,(\upsilon) = \sigma^2\lambda^{-2}\psi'(\lambda^{-2}). \tag{8c}$$

$\psi(\lambda^{-2})$ and $\psi'(\lambda^{-2})$ are the digamma and trigamma functions respectively and their value depends only on λ. (See Lawless [15] for a formal definition of these functions). This implies that $[\psi(\lambda^{-2}) - \ln(\lambda^{-2})] = -0.5772$ and $\psi'(\lambda^{-2}) = \pi^2/6$ when $\lambda = 1$ and that $[\psi(\lambda^{-2}) - \ln(\lambda^{-2})] = 0$ and $\lambda^{-2}\psi'(\lambda^{-2}) = 1$ when $\lambda = 0$.

The advantage of using this distribution is therefore that it encompasses as special cases a variety of well known distributions including, the Exponential, Weibull, Log Normal and Gamma distributions (for $\exp(\upsilon)$). As such a particular type of distribution is not forced upon the data. Rather the data can be used to determine which of all these distributions best describes the frequency with which various prediction errors υ_i will be observed using a particular projection technique (4Θ or 6θ).

The best fitting distribution is defined in terms of the likelihood function, $L(\mu, \sigma, \lambda)$, which measures the joint probability of observing each and every υ_i value. That is, the distribution that best describes the prediction errors is the one with the largest value for $L(\mu, \sigma, \lambda)$. This will correspond to a particular value for λ so that $L(\mu, \sigma, \lambda)$ will be a function of λ. It is often simpler to work with the log likelihood function, so that the distribution that best describes the prediction errors is the one with the largest value for $\ln L(\mu, \sigma, \lambda)$. From Equation 8a

$$\ln L(\mu, \sigma, \lambda) = N[\ln|\lambda| - \ln\Gamma(\lambda^{-2}) - \ln(\sigma) + \lambda^{-2}\ln(\lambda^{-2})] + \sum_{i=1}^{N}\lambda^{-2}(w_i - \exp(\lambda w_1)), \tag{9a}$$

where there are N observations on the prediction error υ and thus the transformation w of a particular creep property.

Values for μ, σ and λ are therefore those values which result in $\ln L(\mu, \sigma, \lambda)$ being maximised. These maximum likelihood estimates are asymptotically efficient and the distributions for μ, σ and λ are also asymptotically normal so that student t tests for statistical significance can be applied to these estimates. Moreover, because all the distributions contained within Equation 8a come from the exponential family, if unbiased minimum variance estimators exist, these maximum likelihood estimates will be them [16]. Numerical optimisation procedures for maximising Equation 9a are described elsewhere [17] and for this paper they were implemented within Excel using its Solver Function.

Once values for μ, σ and λ are obtained in the way described above, they can be substituted into Equation 8b to obtain an estimate of the mean prediction error. A 95% confidence interval for this mean is then found by realising that the square of the ratio of two likelihood functions is asymptotically chi square distributed. Now Equation 9a can be maximised subject to the constraint that the mean error takes on some particular value, say υ^*. If $\ln L(\mu, \sigma, \lambda, \upsilon^*)$ is such a constrained maximised log likelihood and $\ln L(\mu, \sigma, \lambda)$ the unconstrained maximised log likelihood, then a 95% confidence interval for the mean prediction error is made up of all those values for υ^* that result in

$$\Lambda = -2(\ln L(\mu, \sigma, \lambda, \upsilon^*) - \ln L(\mu, \sigma, \lambda)) \tag{9b}$$

being less than 5.02, where 5.02 is the 0.975 percentile of the chi square distribution with one degree of freedom (as there is only one constraint in the constrained maximised log likelihood function).

Similarly, a likelihood ratio statistic to test for a zero mean prediction error can be formed by maximising Equation 9b subject to the constraint that $\upsilon^* = 0$. If $\ln L(\mu, \sigma, \lambda, \upsilon^* = 0)$ is such a constrained maximised log likelihood and $\ln L(\mu, \sigma, \lambda)$ the unconstrained maximised log likelihood then a chi square statistic to test for a zero mean is given by

$$\Lambda_{\upsilon=0} = -2(\ln L(\mu, \sigma, \lambda, \upsilon^* = 0) - \ln L(\mu, \sigma, \lambda)). \tag{9c}$$

$\Lambda_{\upsilon=0}$ is asymptotically chi square distributed with one degree of freedom. As such a value for $\Lambda_{\upsilon=0}$ in excess of 5.02 implies (with 95% certainty) that the mean perdiction error is different from zero, i.e. that a biased prediction has been made.

4. Results

Normal creep curves (curves displaying similar periods of primary and tertiary creep with no inverse primary behaviour) were observed under all the tests carried out. In all cases the 6θ model gave a better fit to the experimental curves than the 4Θ model. Fig. 1 shows a typical example using the experimental data obtained at 773 K and 620 MPa. In Fig. 1a the first 100000 seconds of testing are shown covering early strain up to 1.2%. Over this part of the creep curve the 6θ model fits the experimental data much better than the 4Θ model. Indeed, the predictions only start to deviate systematically from the experimental data for strains below about 0.35%. Fig. 1b shows that for the remaining part of the creep curve both models give very similar degrees of fit to the experimental data. It can therefore be expected that the 6θ model will give similar failure time predictions to those obtained from the 4Θ model.

The interpolation function given by Equation 2b (with $b_{j3} = b_{j4} = 0$ due to the use of a single temperature) related the experimental rate parameter values Θ_2 and Θ_4 to stress very well using the parameter values shown in the first half of Table II. The R^2 values of 70.67% and 89.87% respectively confirm that a large percentage of the variation in these theta vales can be explained by variations in stress. Fig. 2a and b give a

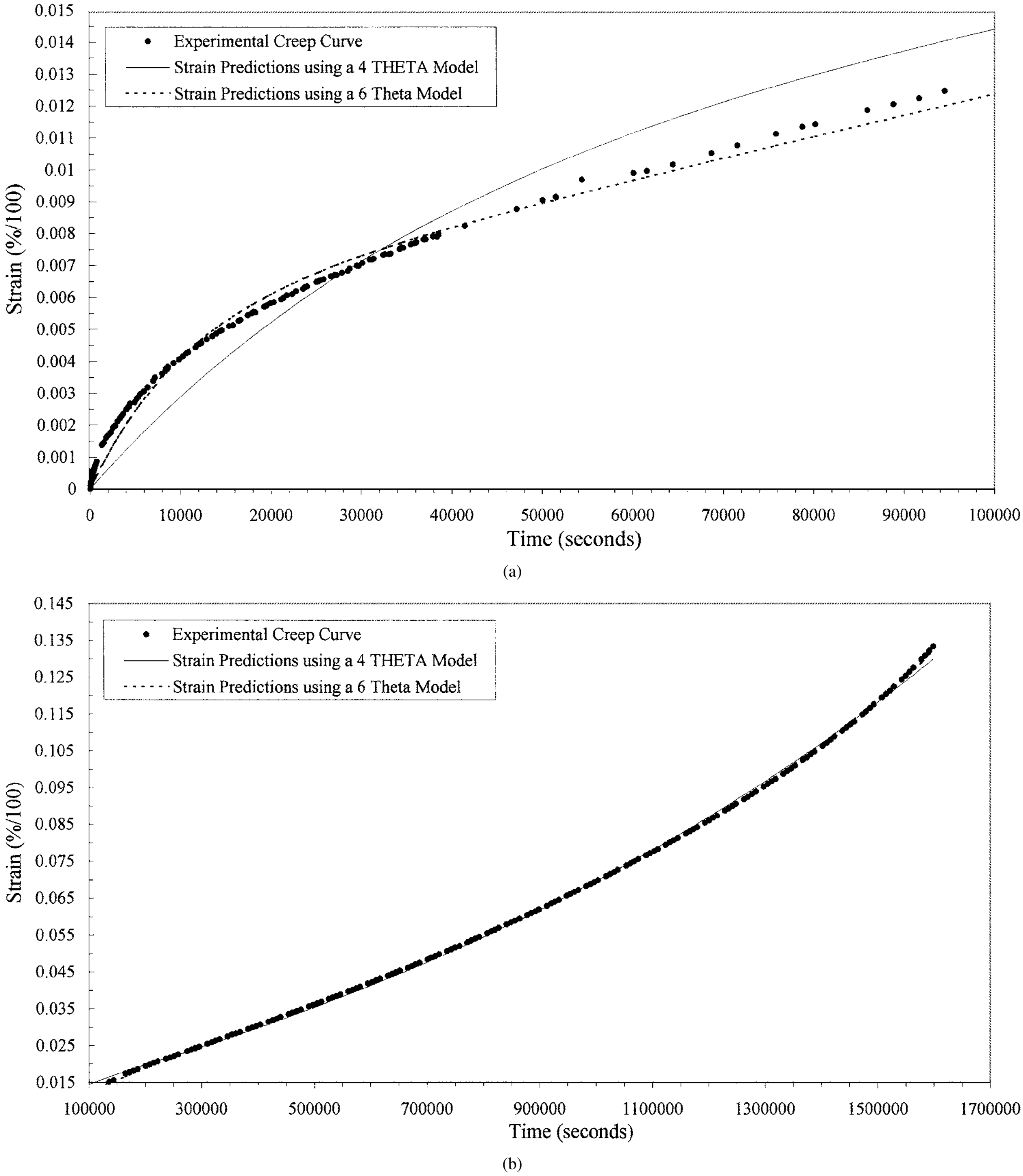
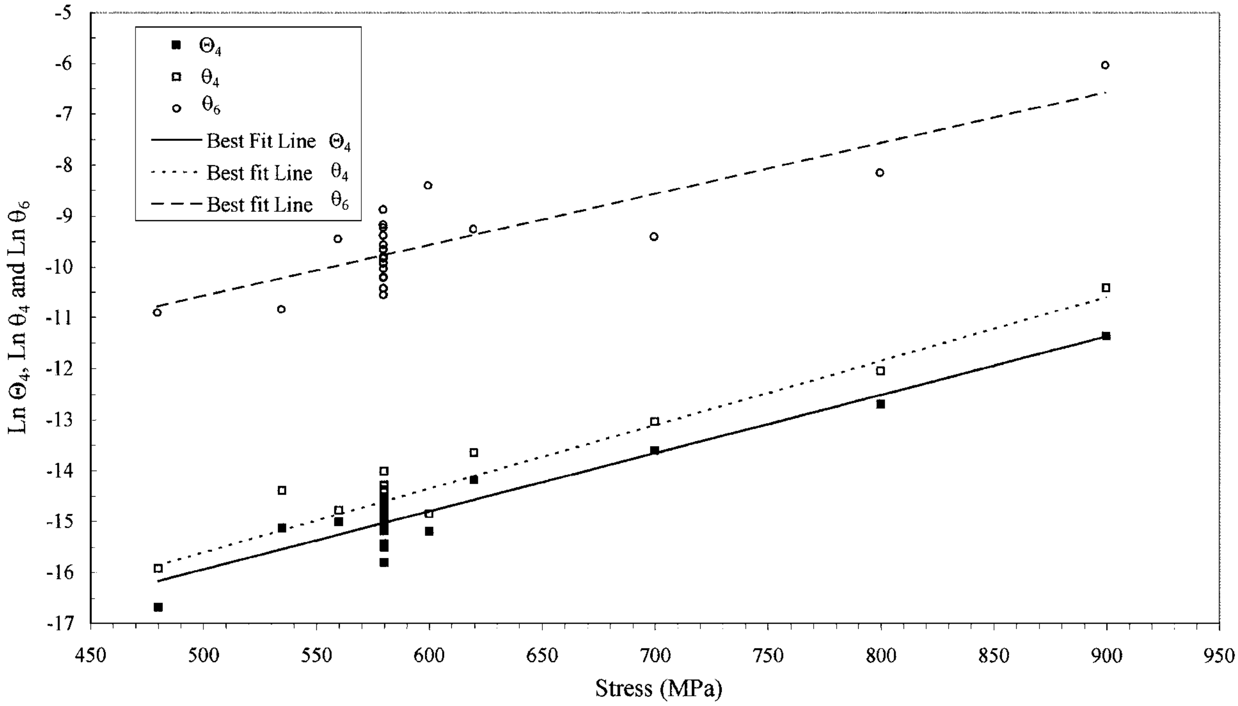


Figure 1 (a) Comparison between actual and predicted creep strains over the first 100000 seconds of testing using the 4Θ and 6θ models at 620 MPa and 773 K. (b) Comparison between actual and predicted creep strains over the last 1600000 seconds of testing using the 4Θ and 6θ models at 620 MPa and 773 K.

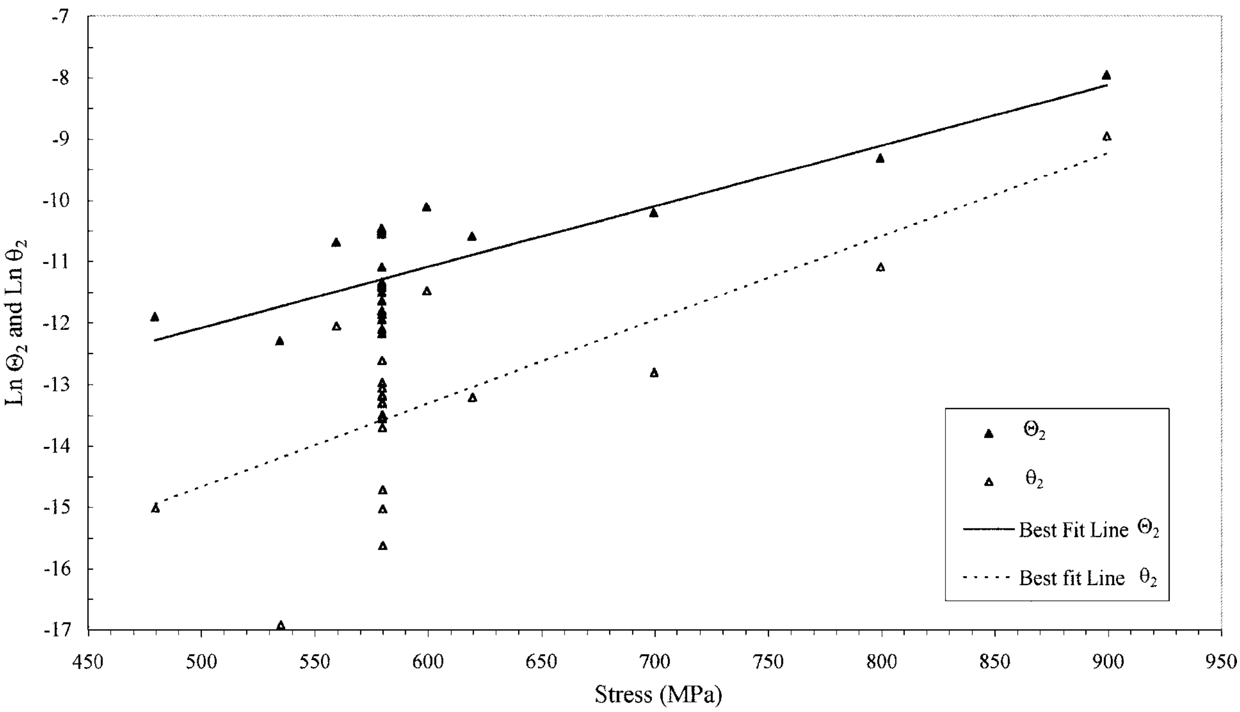
TABLE II Interpolation coefficients in Equation 2b for the 4Θ and 6θ models for Titanium alloy Ti-6.2.4.6 at 773 K

	4Θ			6θ		
	b_{j1}	b_{j2}	R^2 (%)	b_{j1}	b_{j2}	R^2 (%)
Θ_1 (θ_1)	−5.9798	0.00237	58.90	−5.0776	0.00176	2.51
Θ_2 (θ_2)	−17.0439	0.00992	70.67	−21.4866	0.01362	53.32
Θ_3 (θ_3)	−4.8228	0.00326	38.74	−5.1833	0.00225	5.14
Θ_4 (θ_4)	−21.6740	0.01147	89.87	−21.8947	0.01258	89.46
(θ_5)	—	—	—	−5.8677	0.00103	9.95
(θ_6)	—	—	—	−15.6153	0.01008	71.93

R^2 is the coefficient of determination measuring the percentage variation in Θ_j or θ_j explained by variations in stress. b_j are the coefficients in Equation 2b where $b_{j3} = b_{j4} = 0$ as all the above results were obtained at the single temperature of 773 K.



(a)



(b)

Figure 2 (a) The variation of Θ_4, θ_4 and θ_6 with stress at 773 K for Ti 6.2.4.6. (b) The variation of Θ_2 and θ_2 with stress at 773 K for Ti 6.2.4.6.

visual impression of this goodness of fit where much of the unexplained variation is clearly attributable to the fifteen repeat tests at 580 MPa. As expected the fit of Equation 2b to the strain like quantities Θ_1 and Θ_3 is not so good with the R^2 values shown in Table II being 58.9% and 38.74% respectively. (See Fig. 3a and b for a visual impression of this poorer fit).

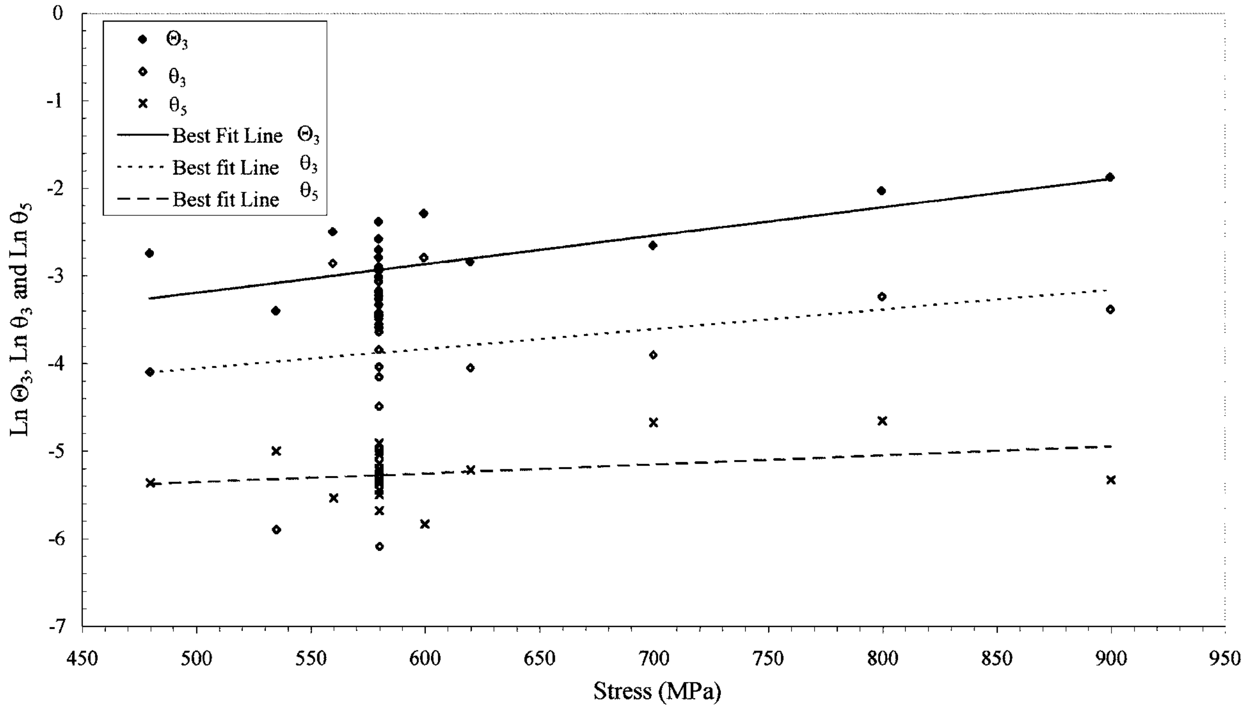
It was not initially clear whether the same interpolation function would be appropriate for the parameters derived from the 6θ model. Fig. 2a and b and the second half of Table II show the fit of Equation 2b to the data θ_2, θ_4 and θ_6 and it is clear that these fits are comparable to those obtained for the rate parameters in the 4θ model. Indeed, the best fit lines for θ_4 and Θ_4 are very similar in terms of both the slope of the best fit line and the R^2 value. The R^2 value for θ_6 is comparable to that on Θ_2 and the R^2 value for θ_2 is in excess of 50%. Again the fit of Equation 2b to the strain like quantities θ_1, θ_3 and θ_5 is not so good as that obtained for the rate parameters—see Fig. 3a and b.

Table III shows that for most of the creep properties studied the prediction errors followed a three parameter log gamma distribution with λ somewhere between 0.1 and 0.5. This implies that the prediction error distribution is skewed to the left. The exceptions to this finding are the prediction error for the minimum creep rate (to be expected given the inverse relationship between failure times and minimum creep rates) and times

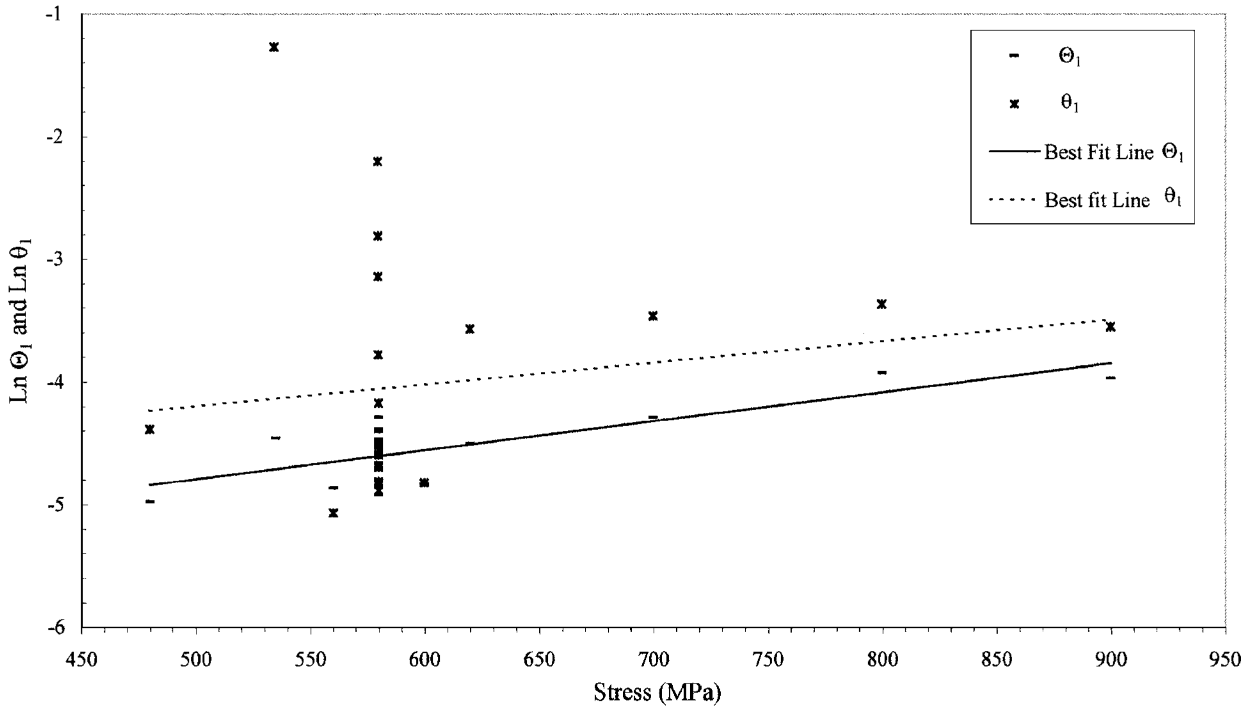
TABLE III Statistical distributions and analysis of errors in interpolation of important creep properties

Creep Property	Model	μ	σ	λ	ln $L(\mu, \sigma, \lambda)$	Mean error, υ	$\Lambda_{\upsilon=0}$
$t_{0.05\%}$	4Θ	−1.826 [−12.59]	0.686 [5.74]	0.4	−24.583	−1.9669	55.29*
	6θ	−1.005 [−7.31]	0.646 [5.65]	0.5	−23.540	−1.1734	37.45*
$t_{0.1\%}$	4Θ	−1.534 [−15.17]	0.485 [5.87]	0.1	−16.013	−1.5582	57.11*
	6θ	−0.741 [−7.76]	0.457 [5.84]	0.2	−14.750	−0.7873	33.54*
$t_{0.2\%}$	4Θ	−1.024 [−12.42]	0.393 [5.80]	0.3	−11.479	−1.0842	52.85*
	6θ	−0.327 [−4.06]	0.385 [5.84]	0.2	−10.830	−0.3656	15.66*
$t_{0.27\%}$	4Θ	−0.822 [−10.41]	0.378 [5.84]	0.2	−10.396	−0.8604	44.07*
	6θ	−0.167 [−2.13]	0.377 [5.87]	0.1	−10.260	−0.1863	5.13
$t_{0.35\%}$	4Θ	−0.610 [−7.55]	0.386 [5.84]	0.2	−10.910	−0.6489	32.59*
	6θ	−0.026 [−0.32]	0.392 [5.87]	0.05	−11.127	−0.0362	0.19
$t_{0.5\%}$	4Θ	−0.301 [−3.62]	0.398 [5.87]	0.05	−11.468	−0.3107	11.11*
	6θ	0.141 [1.71]	0.398 [5.87]	0.01	−11.419	0.1390	2.65
$t_{0.75\%}$	4Θ	−0.053 [−0.63]	0.397 [5.79]	−0.3	−11.728	0.0078	0.01
	6θ	0.038 [0.47]	0.384 [5.86]	0.1	−10.633	0.0186	0.054
$t_{1.0\%}$	4Θ	0.053 [0.69]	0.363 [5.64]	−0.5	−10.301	0.1478	3.55
	6θ	−0.046 [−0.61]	0.365 [5.86]	0.1	−9.479	−0.0641	0.703
$\dot{\varepsilon}_M$	4Θ	0.114 [1.87]	0.291 [5.86]	−0.1	−4.276	0.1284	4.18
	6θ	0.014 [0.24]	0.290 [5.87]	−0.01	−4.157	0.0157	0.068
t_F (using rupture strain)	4Θ	−0.139 [−3.13]	0.212 [5.82]	0.3	2.705	−0.1719	12.48*
	6θ	−0.124 [−2.75]	0.214 [5.82]	0.3	2.472	−0.1564	10.51*
t_F (using Monkman–Grant)	4Θ	−0.085 [−1.87]	0.216 [5.82]	0.3	2.327	−0.1174	6.28*
	6θ	0.018 [0.41]	0.216 [5.81]	0.3	2.259	−0.0144	0.099

$t_{x\%}$ is time to x% strain, t_F is time to failure, $\dot{\varepsilon}_M$ is the minimum creep rate, μ is the location parameter in Equation 8a, σ is the scale parameter in Equation 8a, λ is the shape parameter in Equation 8a, υ is the prediction error as defined by Equation 7b, ln $L(\mu, \sigma, \lambda)$ is the maximised log likelihood value as given by Equation 9a and $\Lambda_{\upsilon=0}$ is the chi square test shown in Equation 9c for a zero mean prediction error.
Student t statistics for the null hypothesis $\mu = 0$ and $\sigma = 0$ are shown in parenthesis.
*Significant at the 5% significance level implying, with 95% certainty, that the mean prediction error is biased, i.e. non zero.



(a)



(b)

Figure 3 (a) The variation of Θ_3, θ_3 and θ_5 with stress at 773 K for Ti 6.2.4.6. (b) The variation of Θ_1 and θ_1 with stress at 773 K for Ti 6.2.4.6.

to relatively high strains using the 4θ model. In these fewer cases λ is always negative implying a skew to the right. Whilst the 95% confidence intervals (as defined using Equation 9b) for λ for all the creep properties contained the log normal and Weibull distributions these are clearly not the distributions most supported by the data. (More details of these confidence intervals are available from the author on request).

Given the non normal nature of most of the creep property prediction errors the standard student t test for the null hypothesis that the population mean prediction error is zero is invalid. Instead the chi square test defined in Equation 9c (using the distribution most supported by the data in Table III) was used to test this null hypothesis. Results are shown in the last column of Table III and in Fig. 4.

Taking first the predictions of time to $x\%$ strain. Using the 4Θ model the null hypothesis was accepted down to a strain of 0.75%. For lower strains the mean prediction error was biased, i.e. significantly different from zero at the 5% significance level. This shows up in Fig. 4a where the 95% confidence intervals do not cross over the zero mean axis below 0.75%. However, when using the 6θ model the null hypothesis could be

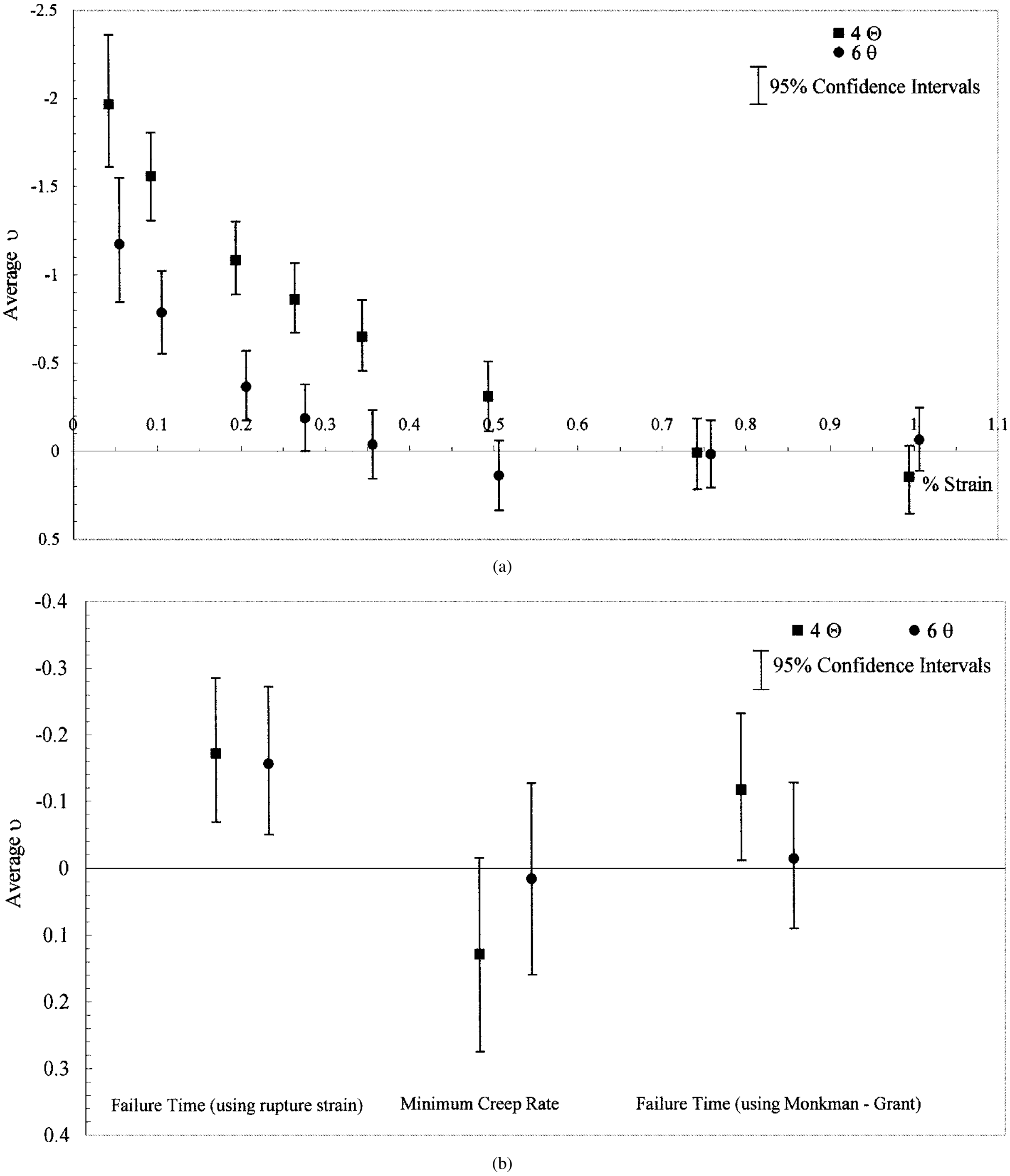


Figure 4 (a) The variation in the mean prediction error for time to $x\%$ strain with $x\%$ strain for the 4Θ and 6θ models together with a 95% confidence interval for such mean error predictions. (b) Mean prediction error for time to failure and the minimum creep rate using the 4Θ and 6θ models together with a 95% confidence interval for such mean error predictions.

accepted down to 0.27% strain—a substantial improvement over the 4Θ model. Whilst for lower strains than this the 6θ model produced biased predictions, Fig. 4a clearly shows that no matter what strain is selected below 0.5% the 6θ model will always produced a prediction that is significantly better than that derived from the 4Θ model. This is shown by the fact that the confidence intervals in Fig. 4a below 0.5% strain do not overlap each other.

Finally, consider the predictions of time to failure shown at the bottom of Table III and in Fig. 4b. When the 4Θ and 6θ models are used in conjunction with a prediction of rupture strain, they yield very similar mean prediction errors for time to failure. Both produce a mean error that is significantly different from zero. However, the 6θ model produces a much better prediction of the minimum creep rate using Equation 5c than the 4Θ model does using Equation 5b. When these creep rate predictions are inserted into the following estimate of the Monkman–Grant relation

$$t_F = \frac{0.387}{\varepsilon_M^{0.914}},$$

the 6θ model unsurprisingly produces superior time to failure predictions. Whilst the 4Θ model in conjunction with the Monkman–Grant relation produces a mean failure time prediction error that is significantly different from zero, the 6θ model with the Monkman–Grant relation produces a zero mean error. This suggests that when using a 6θ model it may be better to incorporate the Monkman–Grant relation into the model when using it to predict a time to failure.

5. Conclusions

A number of conclusions can be drawn from the results shown above. First, the shape of a 6θ creep curve better fits the experimental data for Ti.6.2.4.6 than the shape of a 4Θ creep curve. Second, the errors in the interpolated times to small strains from either model are not best described using a normal or Weibull distribution although neither distribution can be completely rejected by the data. Third, for strains below 0.27% both the 4Θ and 6θ models produce mean values of the errors in interpolation that are not zero. For strains above 0.27% the 6θ model produces mean values of the errors in interpolation that are zero. Using the 4Θ model zero mean interpolation errors do not happen until strains in excess of 0.5% are reached. Fourth, the 6θ model produces zero mean interpolation errors for the minimum creep rate whilst the 4Θ model does not. So when used in combination with the Monkman–Grant relation the 6θ model produces a zero mean interpolation error for the time to failure—the 4Θ model does not.

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